

# Enabling Differentiated Services Using Generalized Power Control Model in Optical Networks

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**Abstract**—This paper considers a generalized framework to study OSNR optimization-based end-to-end link level power control problems in optical networks. We combine favorable features of game-theoretical approach and central cost approach to allow different service groups within the network. We develop solutions concepts for both cases of empty and nonempty feasible sets. In addition, we derive and prove the convergence of a distributed iterative algorithm for different classes of users. In the end, we use numerical examples to illustrate the novel framework.

## I. INTRODUCTION

Reconfigurable optical Wavelength-Division Multiplexing (WDM) communication networks with arbitrary topologies are currently enabled by technological advances in optical devices such as optical add/drop MUXes (OADM), optical cross connects (OXC) and dynamic gain equalizer (DGE). It is important that channel transmission performance and quality of service (QoS) be optimized and maintained after reconfiguration. At the physical transmission level, channel performance and QoS are directly determined by the bit-error rate (BER), which in turn depends on optical signal-to-noise ratio (OSNR), dispersion and nonlinear effects, [1]. Thus, OSNR is considered as the dominant performance parameter in link-level optimization. Conventional off-line OSNR optimization is done by adjusting channel input power at transmitter (Tx) to equalize the dominant impairment of noise accumulation in chains of optical amplifiers. However, for reconfigurable optical networks, where different channels can travel via different optical paths, it is more desirable to implement on-line decentralized iterative algorithms to accomplish such adjustment.

Recently, this problem is addressed in many research works [2],[3],[4], and two optimization-based approaches are prevalently used: the central cost and the non-cooperative game approach. The goals and models of the two approaches are inherently different. Central cost approach satisfies the target OSNR with minimum total power consumption. The model embeds the OSNR requirements in its constraints and indirectly optimizes a certain design criterion. Such model yields a relatively simple closed-form solution; however, it doesn't optimize OSNR in a direct fashion, and thus, channel performance can be potentially improved for users who need higher quality of transmission. On the other hand, the game

approach is a naturally distributed model which directly optimizes OSNR based on a payoff function in a non-cooperative manner. Each user optimizes her own utility to achieve the best possible OSNR. The solution from this approach is given by Nash equilibrium. As a result, this solution concept yields best achievable OSNR levels for each user. Since the game approach involves a cost function arising from pricing, it gives an over-allocation of resources. Some users may wish to avoid such cost and only demand a basic level of transmission. Apparently, these two approaches are for two different type of users and different transmission purposes.

To make use of the advantages from each approach, we propose a generalized model that combines their features. Such a generalization allows to accommodate different types of users and also provides a novel mixed framework to study OSNR power control problem. We separate users into two different categories. One type of users are those who are willing to pay a price to fully optimize their transmission performance. Another type of users are those who are content with basic transmission quality, or OSNR level, set by the network. The quality of service (QoS) can be met for the former by a game-theoretically based optimization approach; and for the later by a mechanism similar to central cost approach.

The contribution of this paper lies in the capability of service differentiation of the generalized model. For simplicity, total capacity constraints are not considered. The paper is organized as follows. In section 2, we review the network OSNR model and the basic concepts about the two optimization-based approaches. In section 3, we establish a general framework and propose two solution concepts for two different cases of feasible sets. Section 4 gives an iterative algorithm to achieve such solutions in the framework. This is illustrated in section 5 by numerical examples. Section 6 concludes the paper and points out future directions of research.

## II. BACKGROUND

### A. Review of Optical Network Model

Consider a network with a set of optical links  $\mathcal{L} = \{1, 2, \dots, L\}$  connecting the optical nodes, where channel add/drop is realized. A set  $\mathcal{N} = \{1, 2, \dots, N\}$  of channels are transmitted, corresponding to a set of multiplexed wavelengths. Illustrated in Figure 1, a link  $l$  has  $K_l$  cascaded optically amplified spans. Let  $\mathcal{N}_l$  be the set of channels transmitted over link  $l$ . For a channel  $i \in \mathcal{N}$ , we denote by  $\mathcal{R}_i$  its optical path, or collection of links, from source (Tx) to destination (Rx). Let  $u_i$  be the  $i$ th channel input optical power (at Tx), and

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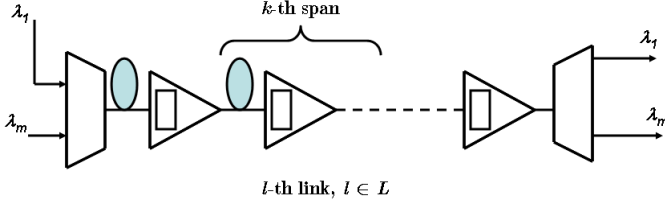


Fig. 1. A Typical Optical Link in DWDM Optical Networks

$\mathbf{u} = [u_1, \dots, u_N]^T$  the vector of all channels' input powers. Let  $s_i$  be the  $i$ th channel output power (at Rx), and  $n_i$  the optical noise power in the  $i$ th channel bandwidth at Rx. The  $i$ th channel optical OSNR is defined as  $OSNR_i = \frac{s_i}{n_i}$ . In [2], some assumptions are made to simplify the expression for OSNR, typically for uniformly designed optical links:

- 1) (A1) Amplified spontaneous emission (ASE) noise power does not participate in amplifier gain saturation.
- 2) (A2) All the amplifiers in a link have the same spectral shape with the same total power target and are operated in automatic power control mode.

Under A1 and A2, dispersion and nonlinearity are considered to be limited, and ASE noise accumulation will be the dominant impairment. The OSNR for the  $i$ th channel is given as

$$OSNR_i = \frac{u_i}{n_{0,i} + \sum_{j \in \mathcal{N}} \Gamma_{i,j} u_j}, i \in \mathcal{N} \quad (1)$$

where  $\mathbf{\Gamma}$  is the full  $n \times n$  system matrix which characterizes the coupling between channels.  $n_{0,i}$  denotes the  $i$ th channel noise power at the transmitter. System matrix  $\mathbf{\Gamma}$  encapsulates the basic physics present in optical fiber transmission and implements an abstraction from a network to an input-output system. This approach has been used in [5] for the wireless case to model CDMA uplink communication. Different from the system matrix used in wireless case, the matrix  $\mathbf{\Gamma}$  given in (2) is commonly asymmetric and is more complicatedly dependent on parameters such as spontaneous emission noise, wavelength-dependent gain, and the path channels take.

$$\Gamma_{i,j} = \sum_{l \in \mathcal{L}} \sum_{k=1}^{K_l} \frac{G_{l,j}^k}{G_{l,i}^k} \left( \prod_{q=1}^{l-1} \frac{\mathbf{T}_{q,j}}{\mathbf{T}_{q,i}} \right) \frac{ASE_{l,k,i}}{P_{o,l}}, \forall j \in \mathcal{N}_l. \quad (2)$$

where  $G_{l,k,i}$  is the wavelength dependent gain at  $k$ th span in  $l$ th link for channel  $i$ ;  $\mathbf{T}_{l,i} = \prod_{q=1}^{K_l} G_{l,k,i} L_{l,k}$  with  $L_{l,k}$  being the wavelength independent loss at  $k$ th span in  $l$ th link;  $ASE_{l,k,i}$  is the wavelength dependent spontaneous emission noise;  $P_{o,l}$  is the output power at each span.

### B. Central Cost Approach

Similar to the SIR optimization problem in the wireless communication networks [6], [7], OSNR optimization achieves the target OSNR predefined by each channel user by allowing the minimum transmission power. Let  $\gamma_i, i \in \mathcal{N}$  be the target OSNR for each channel. By setting the OSNR requirement as a constraint, we can arrive at the following central cost optimization problem (CCP):

$$\begin{aligned} \text{(CCP)} \quad & \min_{\mathbf{u}} \sum_{i \in \mathcal{N}} u_i \\ \text{subject to} \quad & OSNR_i \geq \gamma_i \quad \forall i \in \mathcal{N}. \end{aligned} \quad (3)$$

Under certain conditions, it has been shown in [2] that the feasible set of (CCP) is nonempty and the optimal solution is achievable at the boundary of the feasible set.

The formulated optimization problem can be extended to incorporate more constraints such as

$$u_{i,\min} \leq u_i \leq u_{i,\max}, \quad (4)$$

where  $u_{i,\min}$  is minimum threshold power required for transmission for channel  $i$  and  $u_{i,\max}$  is maximum power channel  $i$  can attain. In the central cost approach, power  $u_i$  are the parameters to be minimized and the objective function is linearly separable. In addition, the constraints are linearly coupled. These nice characteristics in central cost approach leads to a relatively simple optimization problem.

### C. Non-cooperative Game Approach

Let's review the basic game-theoretical model for power control in optical networks without constraints. Consider a game defined by a triplet  $\langle \mathcal{N}, (A_i), (J_i) \rangle$ .  $\mathcal{N}$  is the index set of players or channels;  $A_i$  is the strategy set  $\{u_i \mid u_i \in [u_{i,\min}, u_{i,\max}]\}$ ; and,  $J_i$  is the cost function. It is chosen in a way that minimizing the cost is related to maximizing OSNR level. In [3],  $J_i$  is defined as

$$J_i(u_i, u_{-i}) = \alpha_i u_i - \beta_i \ln \left( 1 + a_i \frac{u_i}{X_{-i}} \right), i \in \mathcal{N} \quad (5)$$

where  $\alpha_i, \beta_i$  are channel specific parameters, that quantify the willingness to pay the price and the desire to maximize its OSNR, respectively,  $a_i$  is a channel specific parameter,  $X_{-i}$  is defined as  $X_{-i} = \sum_{j \neq i} \Gamma_{i,j} u_j + n_{0,i}$ . This specific choice of utility function is non-separable, nonlinear and coupled. However,  $J_i$  is strictly convex in  $u_i$  and takes a specially designed form such that its first-order derivative is linear with respect to  $\mathbf{u}$ .

The solution from the game approach is usually characterized by Nash equilibrium (NE). Provided that  $\sum_{j \neq i} \Gamma_{i,j} < a_i$ , the resulting NE solution is uniquely determined in a closed form by

$$\tilde{\mathbf{\Gamma}} \mathbf{u}^* = \tilde{\mathbf{b}}, \quad (6)$$

where  $\tilde{\Gamma}_{i,j} = a_i$ , for  $j = i$ ;  $\tilde{\Gamma}_{i,j} = \Gamma_{i,j}$ , for  $j \neq i$  and  $\tilde{\mathbf{b}} = \frac{a_i \beta_i}{\alpha_i} - n_{0,i}$ .

Similar to the wireless case [5], we are able to construct iterative algorithms to achieve the Nash equilibrium. A simple deterministic first order parallel update algorithm is:

$$u_i(n+1) = \frac{\beta_i}{\alpha_i} - \frac{1}{a_i} \left( \frac{1}{OSNR_i(n)} - \Gamma_{i,i} \right) u_i(n). \quad (7)$$

As proved in [3], the algorithm (7) converges to Nash equilibrium  $\mathbf{u}^*$  provided that  $\frac{1}{a_i} \sum_{j \neq i} \Gamma_{i,j} < 1, \forall i$ .

## III. GENERALIZED MODEL

In this section, we consider a game designed to allow service differentiation by separating users into two groups: one group seeking a minimum OSNR target and another group participating in a game setting for OSNR optimization. The minimum OSNR for target seekers is set by the network to ensure the minimum quality of service. However, the game

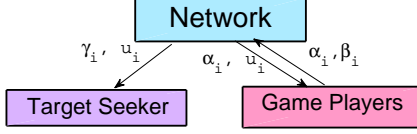


Fig. 2. Game players and target seekers in the network

players can submit their parameters and optimize their service accordingly, but they have to pay a price set by the network for unit power consumption. This concept is illustrated in Figure 2. Let's denote set  $\mathcal{N}_1 = \{1, 2, \dots, N_1\}$  as the set of competitors, i.e. users who wish to compete for an optimal OSNR. Let set  $\mathcal{N}_2 = \{N_1 + 1, \dots, N_2\}$  be the group of users with target OSNR given by  $\gamma_i, i \in \mathcal{N}_2$ . Let  $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2$ ,  $m = |\mathcal{N}_1| = N_1$ ,  $n = |\mathcal{N}_2|$ ,  $N = |\mathcal{N}| = m + n$  and  $\mathbf{u} = [u_1, \dots, u_{N_1}, u_{N_1+1}, \dots, u_{N_2}]^T$ .

For the game-theoretical players, using the cost function given in (5), we can form a system of equations given by

$$a_i u_i + X_{-i} = \frac{a_i \beta_i}{\alpha_i}, \forall i \in \mathcal{N}_1$$

and thus,  $\tilde{\Gamma} \mathbf{u} = \tilde{\mathbf{b}}$ , where  $\tilde{\Gamma} \in \mathcal{R}^{m \times N}$  and  $\tilde{\mathbf{b}} \in \mathcal{R}^m$  are defined as in (6). Users with target OSNR shall have  $\mathbf{u}$  satisfy  $OSNR_i \geq \gamma_i, \forall i \in \mathcal{N}_2$ , or equivalently from (1),

$$\frac{u_i}{\Gamma_{i,i} u_i + \sum_{j \neq i} \Gamma_{i,j} u_j + n_{0,i}} \geq \gamma_i$$

and thus in a matrix form,  $\hat{\Gamma} \mathbf{u} \geq \hat{\mathbf{b}}$ , where  $\hat{\mathbf{b}} = [\gamma_1 n_{0,1}, \dots, \gamma_N n_{0,N}]^T \in \mathcal{R}^n$ ,  $\hat{\Gamma} \in \mathcal{R}^{n \times N}$  and is given in (8). Let  $F_1 = \{\mathbf{u} \in \mathcal{R}^N \mid \tilde{\Gamma} \mathbf{u} = \tilde{\mathbf{b}}\}$  and  $F_2 = \{\mathbf{u} \in \mathcal{R}^N \mid \hat{\Gamma} \mathbf{u} \geq \hat{\mathbf{b}}\}$ . In summary, we have a problem formulated as in (DS), where we find solutions that satisfy  $F_1$  subject to the constraint described by  $F_2$ .

$$\begin{aligned} \text{(DS)} \quad & \tilde{\Gamma} \mathbf{u} = \tilde{\mathbf{b}} \\ \text{s.t.} \quad & \hat{\Gamma} \mathbf{u} \geq \hat{\mathbf{b}} \end{aligned} \quad (9)$$

In the following discussion, we separate (DS) into two cases: (1)  $F = F_1 \cap F_2 \neq \emptyset$ , (2)  $F = F_1 \cap F_2 = \emptyset$ , which require different techniques to find appropriate solutions.

#### A. Non-empty Feasible Set

A non-empty  $F$  may give rise to multiple points that solve (DS). We may impose some design criteria, or, objective function to reformulate DS for finding an appropriate solution that solves DS and meet the design criteria at the same time.

We can use the following result to ensure the nonempty feasible set  $F$ .

**Theorem 3.1:** If  $\bar{\Gamma} = \begin{bmatrix} \tilde{\Gamma} \\ \hat{\Gamma} \end{bmatrix}$  is nonsingular, the feasible set  $F = F_1 \cap F_2$  is non-empty.

*Proof:* Let  $\mu \in \mathcal{R}_+^n$  a nonnegative vector. Equivalently, we can express  $F_2$  into  $F_2 = \{\mathbf{u} \in \mathcal{R}^N \mid \hat{\Gamma} \mathbf{u} = \hat{\mathbf{b}} + \mu, \text{ for some } \mu \in \mathcal{R}_+^n\}$ . The set  $F$  is thus equivalently  $F =$

$\{\mathbf{u} \in \mathcal{R}^N \mid \bar{\Gamma} \mathbf{u} = \phi, \text{ for some } \mu \in \mathcal{R}_+^n\}$ , where  $\bar{\Gamma} = \begin{bmatrix} \tilde{\Gamma} \\ \hat{\Gamma} \end{bmatrix}$  and  $\phi = \begin{bmatrix} \tilde{\mathbf{b}} \\ \hat{\mathbf{b}} + \mu \end{bmatrix}$ . If  $\bar{\Gamma}$  is nonsingular, there exist a unique  $\mathbf{u} \in \mathcal{R}^N$  for every nonnegative  $\mu$ . Therefore  $F$  is non-empty. ■

Suppose conditions in Theorem 3.1 hold and  $F$  is nonempty. We consider an appropriate solution in  $F$  that satisfies a certain design criteria. Thus, we formulate (DSNP<sup>1</sup>) in which we minimize total power consumption subject to the conditions arising from the different service requirements.

$$\begin{aligned} \text{(DSNP)} \quad & \min \sum_i u_i \\ \text{s.t.} \quad & \tilde{\Gamma} \mathbf{u} = \tilde{\mathbf{b}}, \hat{\Gamma} \mathbf{u} \geq \hat{\mathbf{b}} \end{aligned} \quad (10)$$

The constraints of (DSNP) can be relaxed and augmented into

$$\bar{\Gamma} \mathbf{u} \geq \bar{\mathbf{b}}. \quad (11)$$

where  $\bar{\Gamma} = \begin{bmatrix} \tilde{\Gamma} \\ \hat{\Gamma} \end{bmatrix} \in \mathcal{R}^{N \times N}$  and  $\bar{\mathbf{b}} = \begin{bmatrix} \tilde{\mathbf{b}} \\ \hat{\mathbf{b}} \end{bmatrix} \in \mathcal{R}^N$ .

According to the fundamental theorem of linear programming [8], if (DSNP) is realistic, the solution is obtained at the extreme point of the feasible set  $F$ . Since  $F$  has only one extreme point when  $\bar{\Gamma}$  is non-singular, the solution is uniquely given by

$$\mathbf{u} = \bar{\Gamma}^{-1} \bar{\mathbf{b}}. \quad (12)$$

To further characterize the solution  $\mathbf{u}$ , we assume strict diagonal dominance of matrix  $\bar{\Gamma}$  [9], which leads to non-singularity of the matrix and uniqueness of the solution.

**Theorem 3.2:** Suppose OSNR targets  $\gamma_i, i \in \mathcal{N}_2$  are chosen such that  $\gamma_i < \frac{1}{\sum_{j \in \mathcal{N}} \Gamma_{i,j}}, i \in \mathcal{N}_2$ . In addition, parameters  $a_i$  are chosen as  $a_i > \sum_{j \neq i, j \in \mathcal{N}} \Gamma_{i,j}, \forall i \in \mathcal{N}_1$ . The matrix  $\bar{\Gamma}$  is strictly diagonally dominant. And thus, a unique solution to (DSNP) is given by (12).

*Proof:* From the assumption that  $\gamma_i \sum_{j \in \mathcal{N}} \Gamma_{i,j} < 1, i \in \mathcal{N}_2$ , it is apparent that  $\gamma_i < \frac{1}{\Gamma_{i,i}}$  and  $|1 - \gamma_i \Gamma_{i,i}| > \gamma_i \sum_{j \neq i} \Gamma_{i,j}, \forall i \in \mathcal{N}_2$ . In addition,  $a_i > \sum_{j \neq i, j \in \mathcal{N}} \Gamma_{i,j}, \forall i \in \mathcal{N}_1$ . Therefore, matrix  $\bar{\Gamma}$  is strictly diagonally dominant. Using Gershgorin theorem in [9], we conclude that there exists a unique solution to (DSNP). ■

The assumption of strict diagonal dominance in Theorem 3.2 is reasonable because typical values of  $\Gamma_{i,j}$  are found to be on the order of  $10^{-3}$  and desirable levels of OSNR are 20-30dB.

**Remark 3.1:** (DSNP) can be seen as a generalized approach that combines central cost approach in [2] and non-cooperative game approach in [3]. When  $N_1 = \emptyset, N_2 \neq \emptyset$ , (DSNP) reduces to the central cost approach. Similarly, when  $N_1 \neq \emptyset, N_2 = \emptyset$ , (DSNP) reduces to the game-theoretical approach and the given solution is Nash equilibrium accordingly. This framework allows to study two different types of users at the same time.

**Remark 3.2:** We illustrate a two-person (DSNP), where player 1 chooses to compete and optimize his utility and player

<sup>1</sup>DSNP stands for "Differentiated Service N-person Problem".

$$\hat{\Gamma} = \begin{bmatrix} -\gamma_{N_1+1}\Gamma_{N_1+1,1} & \cdots & 1 - \gamma_{N_1+1}\Gamma_{N_1+1,N_1+1} & \cdots & -\gamma_{N_1+1}\Gamma_{N_1+1,N} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ -\gamma_{N-1}\Gamma_{N-1,1} & -\gamma_{N-1}\Gamma_{N-1,2} & \cdots & 1 - \gamma_{N-1}\Gamma_{N-1,N-1} & -\gamma_{N-1}\Gamma_N \\ -\gamma_N\Gamma_{N,1} & -\gamma_N\Gamma_{N,2} & \cdots & \cdots & 1 - \gamma_N\Gamma_{N,N} \end{bmatrix}. \quad (8)$$

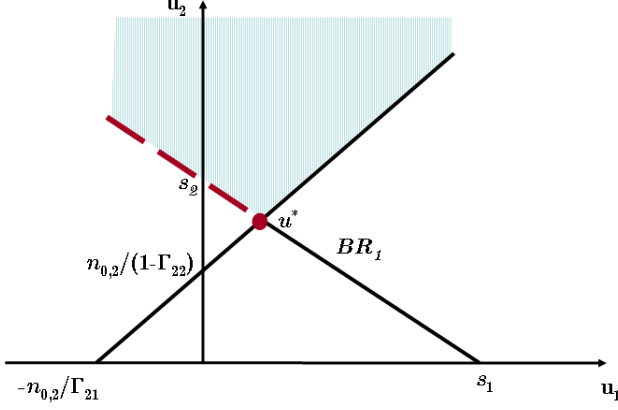


Fig. 3. The feasible set of two-person (DSNP).  $s_1 = \frac{\tilde{b}_1}{a_1}$ ;  $s_2 = \frac{\tilde{b}_1}{\Gamma_{12}}$

2 chooses to meet a certain OSNR target  $\gamma_2$ . We form the 2-by-2 matrix  $\bar{\Gamma}$  and  $\bar{\mathbf{b}}$  as follows.

$$\bar{\Gamma} = \begin{bmatrix} a_1 & \Gamma_{12} \\ -\Gamma_{21}\gamma_2 & 1 - \Gamma_{22}\gamma_2 \end{bmatrix}, \bar{\mathbf{b}} = \begin{bmatrix} \frac{a_1\beta_1}{\alpha_1} - n_{0,1} \\ n_{0,2}\gamma_2 \end{bmatrix}$$

The feasible set  $F = F_1 \cap F_2$  is shown in Figure 3 by a dotted line. The relaxed (DSNP) has its relaxed feasible depicted in the shaded region. The solution is given by  $\mathbf{u}^* = \bar{\Gamma}^{-1}\bar{\mathbf{b}}$ , which is illustrated by the dark point in Figure 3.  $\mathbf{u}^*$  is nonnegative componentwise if network price  $\alpha_1$  is set such that  $s_2 > \frac{n_{0,2}}{1-\Gamma_{22}}$ .

Based on Theorem 3.2, we can further investigate how parameters chosen by game players and target seekers influence the outcome of the allocation. The result is summarized in Theorem 3.3.

**Theorem 3.3:** Let  $\kappa$  be the condition number of  $\bar{\Gamma}$ ,  $T_i = a_i + \sum_{j \neq i, j \in \mathcal{N}} \Gamma_{ij}$ ,  $\forall i \in \mathcal{N}_1$  and  $S_k = 2 - 2\gamma_k\Gamma_{kk}$ ,  $\forall k \in \mathcal{N}_2$ . Suppose  $\bar{\Gamma}$  is strictly diagonally dominant by satisfying conditions in Theorem 3.2. In addition,  $T_i > S_k$  and  $\tilde{b}_i > \tilde{b}_k$ ,  $\forall i \in \mathcal{N}_1, \forall k \in \mathcal{N}_2$ . The maximum allocated power allocated to users are bound as follows.

$$\frac{\max_{i \in \mathcal{N}_2} \gamma_i n_{0,i}}{\max_{i \in \mathcal{N}_1} 2a_i} \leq \|\mathbf{u}\|_\infty \leq \kappa \max_{i \in \mathcal{N}_1} \frac{\beta_i}{\alpha_i}$$

*Proof:* Let  $R_i$  denote the  $i$ -th row absolute sum of matrix  $\bar{\Gamma}$ , i.e.,

$$R_i = \sum_{j \in \mathcal{N}} |\bar{\Gamma}_{ij}|. \quad (13)$$

Using conditions from Theorem 3.2, we arrive at

$$R_i = \begin{cases} 1 + \gamma_i \sum_{j \in \mathcal{N}} \Gamma_{ij} - 2\gamma_i\Gamma_{ii} < 2 - 2\gamma_i\Gamma_{ii}, & i \in \mathcal{N}_2; \\ a_i + \sum_{j \neq i, j \in \mathcal{N}} \Gamma_{ij} < 2a_i, & i \in \mathcal{N}_1. \end{cases} \quad (14)$$

With the assumption that  $a_i + \sum_{j \neq i} \Gamma_{ij} > 2 - 2\gamma_k\Gamma_{kk}$ ,  $\forall i \in \mathcal{N}_1, \forall k \in \mathcal{N}_2$ , we obtain  $\|\bar{\Gamma}\|_\infty = \max_{i \in \mathcal{N}} R_i = \max_{i \in \mathcal{N}} a_i + \sum_{j \neq i} \Gamma_{ij}$ . Using (14) and the fact that  $\Gamma_{ij} \geq 0$ , we obtain an upper and lower bound on  $\|\bar{\Gamma}\|_\infty$ , i.e.,

$$\max_{i \in \mathcal{N}_1} a_i \leq \|\bar{\Gamma}\|_\infty \leq \max_{i \in \mathcal{N}_1} 2a_i. \quad (15)$$

In addition, from  $\tilde{b}_i > \tilde{b}_k$ ,  $\forall i \in \mathcal{N}_1, \forall k \in \mathcal{N}_2$ , we obtain an upper bound and lower bound for  $\|\bar{\mathbf{b}}\|_\infty$ , given by

$$\max_{i \in \mathcal{N}_2} \gamma_i n_{0,i} \leq \|\bar{\mathbf{b}}\|_\infty = \max_{i \in \mathcal{N}} \bar{b}_i \leq \max_{i \in \mathcal{N}_1} \tilde{b}_i = \max_{i \in \mathcal{N}_1} \frac{a_i\beta_i}{\alpha_i} \quad (16)$$

Since  $\bar{\Gamma}$  is strictly diagonally dominant, using matrix norm sub-multiplicativity, we obtain from (12)

$$\frac{\|\bar{\mathbf{b}}\|_\infty}{\|\bar{\Gamma}\|_\infty} \leq \|\mathbf{u}\|_\infty \leq \frac{\kappa \|\bar{\mathbf{b}}\|_\infty}{\|\bar{\Gamma}\|_\infty}, \quad (17)$$

where  $\kappa$  is the condition number of  $\bar{\Gamma}$  given by  $\kappa = \|\bar{\Gamma}\|_\infty \|\bar{\Gamma}^{-1}\|_\infty \geq 1$ .

Using (15), (16) and (17), we obtain

$$\begin{aligned} \frac{\max_{i \in \mathcal{N}_2} \gamma_i n_{0,i}}{\max_{i \in \mathcal{N}_1} 2a_i} \leq \|\mathbf{u}\|_\infty &\leq \frac{\kappa \max_{i \in \mathcal{N}_1} a_i \beta_i / \alpha_i}{\max_{i \in \mathcal{N}_1} a_i} \\ &\leq \frac{\kappa \max_{i \in \mathcal{N}_1} a_i \max_{i \in \mathcal{N}_1} \beta_i / \alpha_i}{\max_{i \in \mathcal{N}_1} a_i} \\ &\leq \kappa \max_{i \in \mathcal{N}_1} \frac{\beta_i}{\alpha_i}. \end{aligned} \quad (18)$$

It is easy to observe that the upper bound is dependent on the parameters of the game players and the lower bound is dependent on the OSNR levels of target seeker and parameter  $a_i$  of the game players. In essence, game players control the outcome of the model and the choice of OSNR target can only affect the lower bound. Such relation describes a fair scenario in which game players, who pay for their power at  $\alpha_i$ , have their choices of parameters  $a_i, \beta_i$  to influence the network allocation.

**Remark 3.3:** Since  $\|\mathbf{u}\|_\infty \leq \|\mathbf{u}\|_2 \leq \sqrt{N}\|\mathbf{u}\|_\infty$ , we can translate the result obtained in (18) directly into Euclidean norm, i.e.,

$$B_\infty^L \leq \|\mathbf{u}\|_2 \leq \sqrt{N}B_\infty^U \quad (19)$$

where  $B_\infty^U = \kappa \max_{i \in \mathcal{N}_1} \frac{\beta_i}{\alpha_i}$  and  $B_\infty^L = \frac{\max_{i \in \mathcal{N}_2} \gamma_i n_{0,i}}{\max_{i \in \mathcal{N}_1} 2a_i}$ . By (19), we can see that the network can encourage uniform channel power distribution by letting  $B_\infty^U$  close to  $\sqrt{N}B_\infty^L$  and provide incentive for differentiated services by letting them far apart. It can be implemented by the network by adjusting OSNR level  $\gamma_i$  and pricing  $\alpha_i$ . Decreasing  $\alpha_i$  encourages more users to be game players, giving rise to more competitions or service differentiation as a result of higher upper bound. On the other hand, increasing  $\gamma_i$  raises the lower bound and encourages more users being target-seekers.

### B. Empty Feasible Set

In this section, we consider the second case where feasible set  $F$  is empty. Instead of finding an appropriate feasible solution, we find the closest points between set  $F_1$  and  $F_2$ . We use a quadratic program (DS2) to minimize the error norm subject to the constraint described by  $F_2$ .

$$\begin{aligned} \text{(DS2)} \quad & \min_{\mathbf{u}} \|\tilde{\mathbf{\Gamma}}\mathbf{u} - \tilde{\mathbf{b}}\|_2 \\ \text{s.t.} \quad & \hat{\mathbf{\Gamma}}\mathbf{u} \geq \hat{\mathbf{b}} \end{aligned} \quad (20)$$

We can turn the constrained problem (20) into an unconstrained problem by studying its corresponding dual problem. Since  $\|\tilde{\mathbf{\Gamma}}\mathbf{u} - \tilde{\mathbf{b}}\|_2 = \mathbf{u}^T \tilde{\mathbf{\Gamma}}^T \tilde{\mathbf{\Gamma}} \mathbf{u} - 2(\tilde{\mathbf{b}}^T \tilde{\mathbf{\Gamma}}) \mathbf{u} + \tilde{\mathbf{b}}^T \tilde{\mathbf{b}}$ , we denote  $\mathbf{H} = \frac{1}{2} \tilde{\mathbf{\Gamma}}^T \tilde{\mathbf{\Gamma}}$ ,  $\mathbf{d} = -2(\tilde{\mathbf{b}}^T \tilde{\mathbf{\Gamma}})$ ,  $\mathbf{D} = -\hat{\mathbf{\Gamma}}(\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \hat{\mathbf{\Gamma}}^T$ ,  $\mathbf{c} = \hat{\mathbf{b}} + \hat{\mathbf{\Gamma}}(\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{d}$ ; and form a Lagrangian from the original problem (DS2).

$$\begin{aligned} D(\mu) &= \min_{\mathbf{u}} \mathcal{L}(\mathbf{u}, \mu) \\ &= \min_{\mathbf{u}} \left( \frac{1}{2} \mathbf{u}^T \mathbf{H} \mathbf{u} + \mathbf{d}^T \mathbf{u} + \tilde{\mathbf{b}}^T \tilde{\mathbf{b}} + \mu^T (-\hat{\mathbf{\Gamma}} \mathbf{u} + \hat{\mathbf{b}}) \right) \end{aligned} \quad (21)$$

Since the objective function is convex, the necessary and sufficient condition for a minimum is that the gradient must vanish, i.e.,

$$\mathbf{H} \mathbf{u} + \mathbf{d} - \hat{\mathbf{\Gamma}}^T \mu = 0. \quad (22)$$

For  $n < N$ ,  $\tilde{\mathbf{\Gamma}}$  is not full rank. Therefore,  $\mathbf{H}$  is singular and there exist multiple solutions to (22). Using pseudoinverse [9], we can find a solution to (22) given by

$$\mathbf{u} = -(\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T (\mathbf{d} - \hat{\mathbf{\Gamma}}^T \mu).$$

Thus, after replacing into (21), we obtain  $\mu$  as a solution to the dual problem (DDS2).

$$\text{(DDS2)} \quad \max_{\mu \geq 0} \frac{1}{2} \mu^T \mathbf{D} \mu + \mu^T \mathbf{c} - \frac{1}{2} \mathbf{d}^T (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{d} + \tilde{\mathbf{b}}^T \tilde{\mathbf{b}} \quad (23)$$

The problem (LDS2) and dual problem (DDS2) can be solved using unconstrained optimization algorithms in [10], [8].

### IV. ITERATIVE ALGORITHM

In this section, we develop algorithm for the case of nonempty  $F$  set. Let  $u_i(n)$  denote the power at channel  $i$  at step  $n$ . An iterative algorithm is given as follows.

$$\begin{cases} u_i(n+1) = \frac{\beta_i}{\alpha_i} - \frac{1}{a_i} \left( \frac{1}{\text{OSNR}_i(n)} - \Gamma_{i,i} \right) u_i(n), & \forall i \in \mathcal{N}_1; \\ u_i(n+1) = \frac{\gamma_i}{1 - \gamma_i \Gamma_{i,i}} \left( \frac{1}{\text{OSNR}_i(n)} - \Gamma_{i,i} \right) u_i(n), & \forall i \in \mathcal{N}_2. \end{cases} \quad (24)$$

**Theorem 4.1:** Algorithm (24) converges provided that  $a_i > \sum_{j \neq i, j \in \mathcal{N}} \Gamma_{i,j}$  and  $\gamma_i$  is chosen such that  $\gamma_i < \frac{1}{\sum_{j \in \mathcal{N}} \Gamma_{i,j}}$ .

*Proof:* We use a similar approach from [3] to show the convergence of (24). Let's define  $e_i(n) = u_i(n) - u_i^*$ , where  $u_i^*$  is given in (12). Since  $\tilde{\mathbf{\Gamma}} \mathbf{u}^* = \tilde{\mathbf{b}}$ ,  $\tilde{\mathbf{\Gamma}}_{i,i} u_i^* + \sum_{j \neq i} \tilde{\mathbf{\Gamma}}_{i,j} u_j^* = \tilde{b}_i$ , for  $i \in \mathcal{N}_1$ ; and,  $\hat{\mathbf{\Gamma}}_{i,i} u_i^* + \sum_{j \neq i} \hat{\mathbf{\Gamma}}_{i,j} u_j^* = \hat{b}_i$ , for  $i \in \mathcal{N}_2$ .

Substitute the expression for  $u_i^*$  into  $e_i(n+1)$ , and we obtain  $e_i(n+1) = u_i(n+1) - u_i^* = -\frac{1}{a_i} \left[ \sum_{j \neq i} \Gamma_{i,j} (u_j(n) - u_j^*) \right]$ ,

for  $i \in \mathcal{N}_1$ ; and  $e_i(n+1) = u_i(n+1) - u_i^* = \frac{1}{1 - \Gamma_{i,i} \gamma_i} \left[ \sum_{j \neq i} \Gamma_{i,j} \gamma_j (u_j(n) - u_j^*) \right]$ , for  $i \in \mathcal{N}_2$ . Let  $\mathbf{e} = [e_i(n)], i \in \mathcal{N}$ . Therefore, for  $i \in \mathcal{N}_1$ ,

$$|e_i(n+1)| = \left| \frac{1}{a_i} \left[ \sum_{j \neq i, j \in \mathcal{N}} \Gamma_{i,j} (e_j(n)) \right] \right| \quad (25)$$

$$\leq \frac{1}{a_i} \sum_{j \neq i, j \in \mathcal{N}} \Gamma_{i,j} \max_{j \in \mathcal{N}} |e_j(n)| \quad (26)$$

$$\leq \frac{1}{a_i} \sum_{j \neq i, j \in \mathcal{N}} \Gamma_{i,j} \|\mathbf{e}(n)\|_{\infty}. \quad (27)$$

and similarly, for  $i \in \mathcal{N}_2$ ,

$$|e_i(n+1)| = \left| \frac{1}{1 - \Gamma_{i,i} \gamma_i} \left[ \sum_{j \neq i, j \in \mathcal{N}} \Gamma_{i,j} \gamma_j (e_j(n)) \right] \right| \quad (28)$$

$$\leq \frac{\gamma_i}{|1 - \Gamma_{i,i} \gamma_i|} \sum_{j \neq i, j \in \mathcal{N}} \Gamma_{i,j} \max_{j \in \mathcal{N}} |e_j(n)| \quad (29)$$

$$\leq \frac{\gamma_i}{|1 - \Gamma_{i,i} \gamma_i|} \sum_{j \neq i, j \in \mathcal{N}} \Gamma_{i,j} \|\mathbf{e}(n)\|_{\infty}. \quad (29)$$

Since we assumed that  $a_i > \sum_{j \neq i, j \in \mathcal{N}} \Gamma_{i,j}$  and  $\gamma_i$  is chosen such that  $\gamma_i < \frac{1}{\sum_{j \in \mathcal{N}} \Gamma_{i,j}} \leq \frac{1}{\Gamma_{i,i}}$ , we can conclude that  $\|\mathbf{e}(n)\| \rightarrow 0$  from the contraction mapping theorem. As a result, we have  $u_i(n) \rightarrow u_i^*$  as  $n \rightarrow \infty$ , for  $i \in \mathcal{N}$ . ■

**Remark 4.1:** From the proof, we note that the rate of convergence of is determined by

$$\sigma = \max \left\{ \max_{i \in \mathcal{N}_1} \frac{\sum_{j \neq i, j \in \mathcal{N}} \Gamma_{i,j}}{a_i}, \max_{i \in \mathcal{N}_2} \frac{\sum_{j \neq i, j \in \mathcal{N}} \Gamma_{i,j} \gamma_j}{1 - \Gamma_{i,i} \gamma_i} \right\}.$$

In addition, it is easy to observe that the OSNR target-seeking users are algorithmically equivalent to competition seeking users by letting  $\beta_i/\alpha_i = 0$  and  $a_i = \Gamma_{i,i} - \frac{1}{\gamma_i}$ ,  $i \in \mathcal{N}_2$ . This is because no notion of pricing is used for the OSNR target seekers and they just have a utility target to meet or equivalently optimize by letting  $a_i = \Gamma_{i,i} - \frac{1}{\gamma_i}$ .

### V. NUMERICAL EXAMPLES

In this section, we illustrate the concept by a MATLAB simulation. We consider an end-to-end link described in Figure 1 with 5 amplified spans. We assume channels are transmitted at wavelengths distributed centered around 1555nm with channel separation of 1nm. Suppose input noise power is 0.5 percent of the input signal power. The gain profile for each amplifier is identically assumed to be parabolic as in Figure 4, which is normalized with respect to  $G_{\max} = 30.0\text{dB}$ . Suppose 20dB is the target OSNR level for users who just want to meet a satisfactory level of transmission. We first show the case of 3 users, in which 2 users need better quality of service and one user is simply interested in meeting 20dB as a target. From Figure 5, we can observe that users who need better services reach an OSNR of 26.33dB and 29.20dB, respectively. With an appropriate choice of initial conditions, the algorithm quickly converges in 1-2 steps. In Figure 6, we similarly show the case of 30 users, in which 20 are game players and 10 are target seekers.

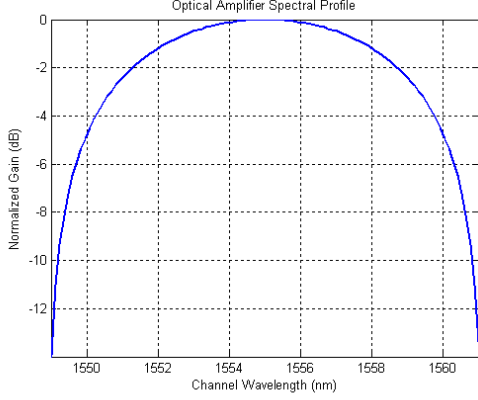


Fig. 4. Optical Amplifier Spectral Profile

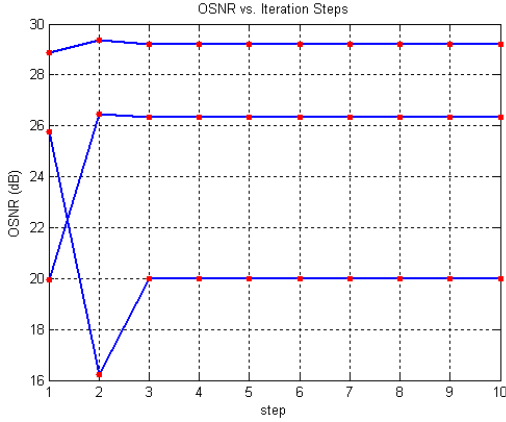


Fig. 5. OSNR simulation with 3 users in time steps

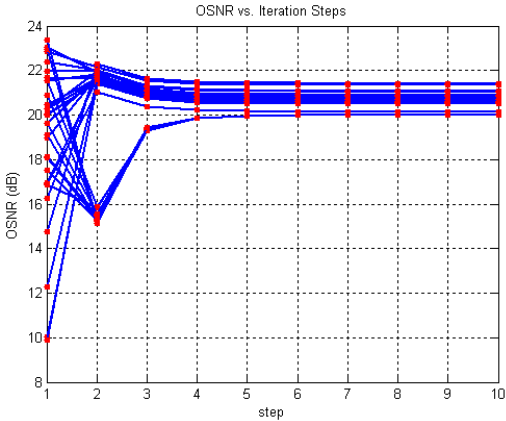


Fig. 6. OSNR simulation with 30 users in time steps

## VI. CONCLUSION

In this paper, we examined a generalized power control model in optical networks, which combines features of central cost approach and game-theoretical approach. It enables two major service types in the network. One is game player, who pays for his power consumption and the other is target seeker, who is satisfied with a minimum service level set by the network. We discussed two different solutions concepts for nonempty and empty feasible set respectively and specifically designed an iterative algorithm that converges to a unique solution for the case of nonempty feasible set. The convergence of the algorithm was proved and illustrated by numerical examples of a WDM end-to-end optical link.

In this work, we didn't include capacity constraints for the sake of simplicity. We hope this work will lead to future investigations of more complicated cases where network constraints and nonlinear effects are considered. In addition, we expect this framework to be used to solve similar problems in other types of networks, for example, wireless networks.

## REFERENCES

- [1] G. Agrawal, *Lightwave Technology*. Wiley-Interscience, 2005.
- [2] L. Pavel, "OSNR optimization in optical networks: Modeling and distributed algorithms via a central cost approach," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 4, pp. 54–65, April 2006.
- [3] —, "A noncooperative game approach to OSNR optimization in optical networks," *IEEE Transactions on Automatic Control*, vol. 51, no. 5, pp. 848–852, May 2006.
- [4] Y. Pan and L. Pavel, "OSNR optimization in optical networks: Extension for capacity constraints," *Proceedings of 2005 American Control Conference*, pp. 2379–2385, June 2005.
- [5] R. Srikant, E. Altman, T. Alpcan, and T. Basar, "CDMA uplink power control as noncooperative game," *Wireless Networks*, vol. 8, p. 659 690, 2002.
- [6] C. Saraydar, N. Mandayam, and D. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Transactions on Communications*, vol. 50, no. 2, pp. 291–414, February 2002.
- [7] C. Saraydar and D. Goodman, "Pricing and power control in a multicell wireless data network," *IEEE Journal of Selected Areas of Communications*, vol. 19, no. 10.
- [8] D. Bertsekas, *Nonlinear Programming*. Athena Scientific, 2003.
- [9] R. Horn and C. Johnson, *Matrix Analysis*. Cambridge University Press, 1990.
- [10] M. Bazaraa, H. Sherali, and C. Shetty, *Nonlinear Programming: Theory and Algorithms*, 2nd ed. Wiley, 1993.

